Visual Electronics: Graphic Analysis of Circuits
(CDT-30)

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Abstract

Electronic circuits are particularly challenging as a consequence of the intrinsically involved non-linearities. While numerical approaches have established themselves along the last decades, graphical analysis methods still provide a great means for better understanding electronic circuit configuration and behavior. Not only the whole solution space can be appreciated, but also the effect of the involved non-linearities can be readily understood. The present work provides a description of the graphical method for circuit analysis and solution based on the concept of characteristic curve. Starting from the short and open connections, and respective curves, other devices are progressively introduced and discussed. Then, a solution approach is presented that involves finding the intersection between two characteristic curves. The superimposition principle, as well as Thevenin and Norton theorems, are also briefly described given their importance in obtaining simplified circuit equivalents that can be used in graphical analysis. Though developed with respect to ideal linear components, the potential of the graphic method for studying non-linear electronic circuits is also illustrated with respect to non-linear resistors and diodes.

> “ Il ponte non è sostenuto da questa o da quella pietra – risponde Marco, – ma dalla linea dell’arco che esse formano. ”
>
> *Italo Calvino, Le Città Invisibili.*

1 Introduction

Electronics circuits are characterized by non-linearities intrinsic to the available devices, such as diodes and bipolar junction transistors. These non-linearities substantially complicate analytical studies of circuit behavior, implying most approaches to become numerically-founded. Yet, graphical analysis of circuits provides interesting complementary means for better understanding the involved devices and their electric/electronic interrelationship. (e.g. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]).

The present work aims at providing a concise, objective, and didactic introduction to the application of the graphical method based on characteristic curves describing the voltage × current behavior of the involved ideal components. The author’s particular motivation and context while preparing this work refers to respective applications of the considered graphic method in developing models for semiconductor devices, such as bipolar junction and MOS transistors and circuits (e.g. [12, 13, 14, 15]). Therefore, the described graphical method is also illustrated with respect to some two-terminal non-linear devices, including non-linear resistors and diodes.

In order to provide a more intuitive understanding of the characteristic curve graphical method, special effort was invested in formulating this method in terms of ‘*interfacing* voltage and current, which provide a kind of ‘plug’ through which the two parts involved in the graphical solution need to be combined while respecting the voltage and current orientation. It is hoped that this approach will contribute to further motivate the consideration of the graphical method by students and researcher.

It should be emphasized that the presented concepts and methods *refer only to ideal circuits, devices and components, so that any application to real-world circuits of any kind is not immediate, demanding great care, experience, and adaptations* required by additional limitations, characteristics and operation of real-world systems.

2 Kirchhoff’s Laws

Electric circuits are, to a great extent, explained by Kirchhoff’s fundamental voltage and current laws, illustrated in

Figure 1 (a) and (b), respectively.

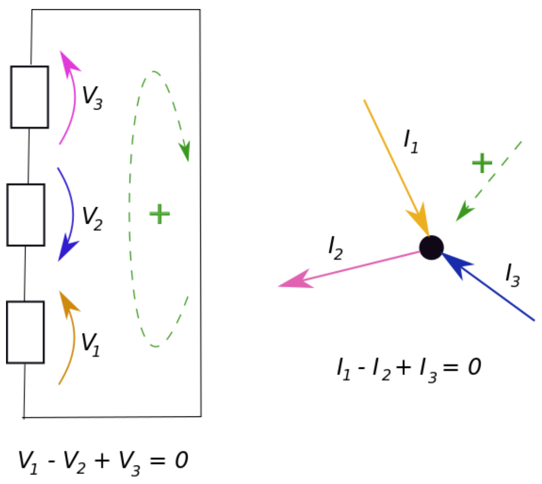
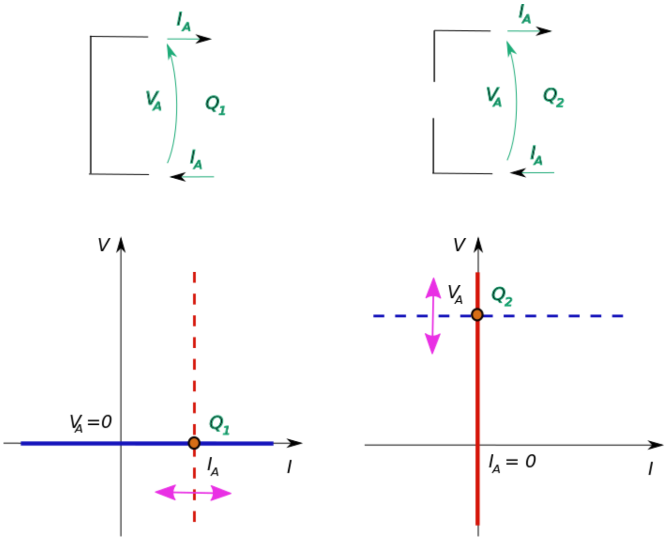


Figure 1: Kirchhoff's laws. The sum of voltages across circuit components (sources or resistors) along a closed mesh and considering a consistent direction (clockwise in this example) should add to zero (a). The sum of currents entering or leaving a node considering their signs, should add to zero (b).



(a) (b)

Figure 2: The ideal short and open connections and their electrical characterization in terms of voltage and current. An ideal short connection will imply no voltage ($V_A = 0$) at any current. An ideal open connection will allow any voltage at zero current ($I_A = 0$).

Kirchhoff's voltage law, illustrated in Figure 1 states that the sum of voltages across every device along a closed circuit mesh, according to a consistently adopted direction (clock- or anti-clockwise) will necessarily add to zero.

Kirchhoff's current law, illustrated in Figure 1 states that the sum of currentss across every device along a closed circuit mesh, according to a consistently adopted direction (in or out the not), will necessarily add to zero. This law relates to flow conservation at the nodes.

3 Ideal Short and Open Connections

The two most basic circuit components are arguably the ideal *short* and *open* connections, illustrated in Figure 2.

By 'ideal' short connection it is meant that it corresponds to full connectivity, no current limits (i.e. withstand any current), with no capacitance and no inductance. Similarly, an ideal open connection implies perfect insulation, no with voltage limit (i.e. withstand any voltage), no capacitance and no inductance. There are no such devices in the real-world, only limited approximations.

Figure 2 also includes the voltage and current at the extremities of the connections, which need to be specified before drawing the respective graphic representation. The orientations of any of these quantities is arbitrary but need to agree and be kept constant during the analysis. The ideal short connection is therefore characterized by necessarily having $V_A = 0$, while the ideal open connection implies $I_A = 0$.

Also show in Figure 2 are the respective *characteristic curves* expressing the corresponding relationship between voltage and current implied by each device.

Observe that the *operation points* Q_1 (open connection) and Q_2 (short connection) are determined by the external circuit (not shown). This fixes the point of operation of these circuits. For instance, if a current I_A leaves the short connection in Figure 2, it will determine the respective operation point $(I = I_A, V = 0)$. In this case, the operation point will follow the imposed current I_A. A similar situation arises for the ideal open connection, with the voltage V_A defining the respective operation point $(I = 0, V_A)$.

As we will see in Section 4, the ideal short and open connections can be understood as particular cases of an ideal *resistor* or even of ideal voltage and current sources when set to zero output.

4 Ohm's Law: The Ideal Resistor

Figure 3 illustrates an *ideal* resistor, which has no power limitation (i.e. any current or voltage), no capacitance, and no inductance. Only approximations of this device exist in the real world, being power-limited (i.e. limited current and voltage) and exhibiting non-null capacitance and inductance.

In the case of this specific device, the current and voltage at its extremities necessarily follow *Ohm's Law*, which basically states that the voltage across a resistor is directly proportional, through the constant R, to the respective current.

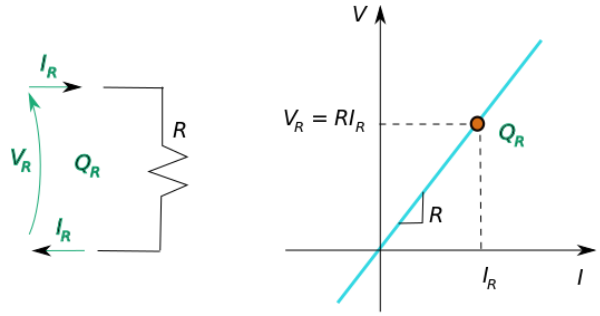


Figure 3: The electrical device known as a *resistor* has its operation defined by Ohm's Law $V = RI$. The respective characteristic curve representation corresponds to a straight line that passes through the origin of the coordinate system.

Thus, if we apply a current I_R, a respective voltage V_R will be implied by Ohm's law. Similarly, applying a voltage V_R, a respective current I_R will be obtained. Observe that the voltage direction is *opposite* to that of the respective current along the device.

As $R \to 0$, the resistor tends to become a short connection, with the inclination of the respective straight characteristic curve tending to zero while still crossing the coordinates origin, approaching a short connection. Similarly, if we make $R \to \infty$, the characteristic line will become vertical, while still crossing the coordinates origin, and the resistor will approach an open connection.

It is also interesting to consider the effect of changing the directions of the interface voltage and current in the characteristic curve representation, which is illustrated and explained in Figure 4.

5 Electrical Power

In addition to voltage and current, electrical power (measure in Watts) constitutes another especially important property and aspect of electrical circuits. The power that a resistor with resistance R dissipates as a consequence of being subjected to current I and voltage V is given as:

$$P = |VI| = \frac{V^2}{R} = RI^2 \qquad (1)$$

In practical situations, it is very important never to submit a resistor to situations nearing or, even worse, exceeding its power dissipation, which involves production of heat. It is also a good practice to allow for wide safety margins.

Given an electric dipole with interfacing voltage and current equal to V_A and I_A, the power that will be dissipated corresponds to $P_A = V_A I_A$. Thus, it is possible to include in the graphical characteristic curve another curve indicating the limit of maximum power dissipation P_{max} expected. This maximum power curve can be drawn as $V = P_{max}/|I|$.

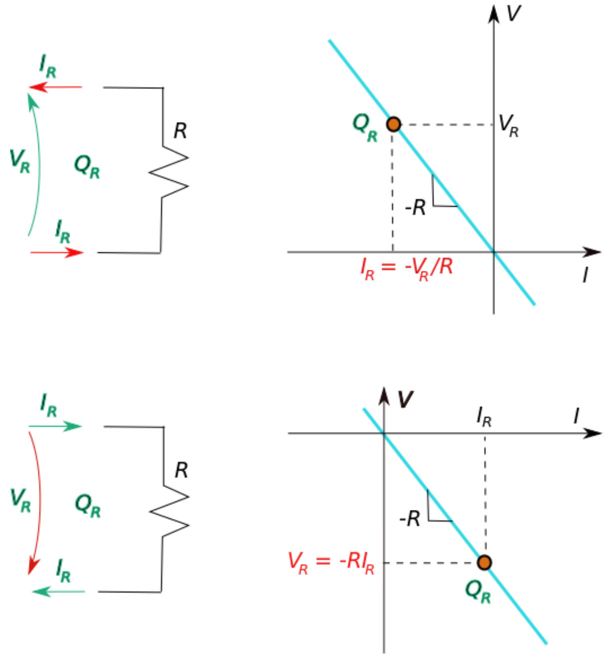


Figure 4: If the current direction of the reference configuration shown in Fig. 3 is inverted (a), the characteristic curve representation should have the curve inverted with respect to the current axis. An analogous situation is defined with respect to the inversion of the voltage direction, shown in (b).

Figure 5 illustrates the ideal resistor and its respective characteristic curve representation including a hypothetical maximum power limitation curve defined as $V = P_{max}/|I|$.

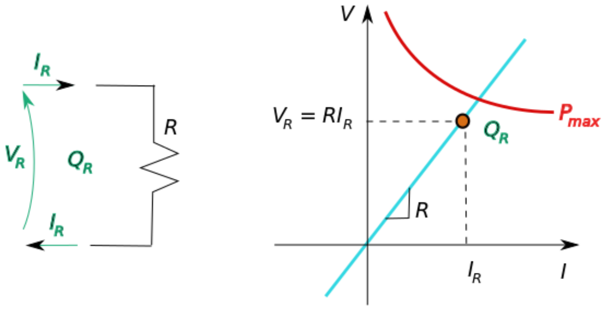
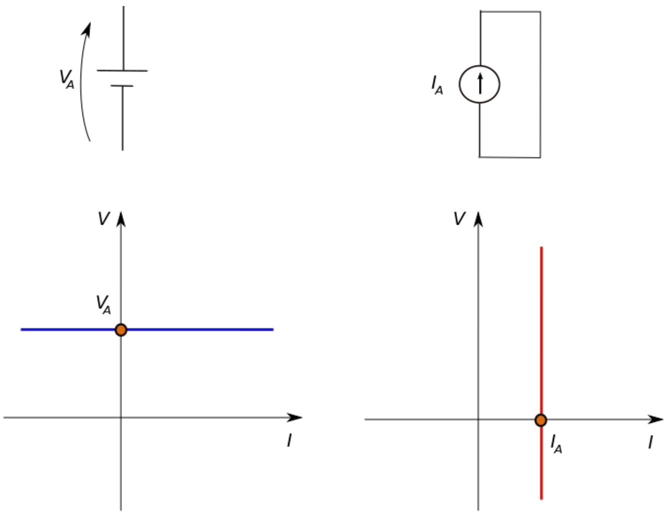


Figure 5: The resistor and its respective characteristic curve incorporating the curve of maximum power dissipation (red). The operation of the circuit should never be allowed near or above the maximum power curve in the respective space (V, I).

6 Ideal Voltage and Current Sources

Two other important devices in electrical circuits are the *ideal voltage and current sources*, depicted in Figure 6.



(a) (b)

Figure 6: The ideal voltage and current sources. The former can supply any current at fixed voltage, while the latter can supply any voltage at fixed current. Observe that the ideal voltage source is shown in open circuit, with respective reference operation point Q_1 at $(I = 0, V = V_A)$. The ideal current source is shown in closed circuit, setting the reference operation point Q_2 at $(I = I_A, V = 0)$.

The ideal voltage source is capable of keeping the voltage at its terminals constant while delivering any current (even reverse, which is not the case in typical real-world sources), a hypothetical situation that would imply infinite power delivery, hence the name 'ideal'. When kept in open circuit as in Figure 6(a), the ideal voltage source will deliver no current and therefore be at the operation point $(I = 0, V = V_A)$.

The ideal current source supplies fixed current whenever the voltage (even reversed, which is not the case in typical real-world sources) at its terminals, which would imply in the theoretical situation of infinite power delivery, hence its name 'ideal'. When in closed loop as in in Figure 6, the ideal voltage source will be a null voltage, and therefore be at the operation point $(I = I_A, V = 0)$.

In typical ideal circuit configurations, both the ideal voltage and current sources will typically be at non-zero current and voltage, as indicated in Figure 7.

Observe that these two ideal sources are shown in Figure 7 considering interaction with some external circuitry (generic, not shown) through *interfacing* voltage V_A and current I_A.

The ideal voltage and current source can also be *adjustable*, as shown in Figure 8. Observe that when an ideal voltage source is set to supply $0V$, it will correspond electrically to an ideal short connection: compare Figure 8(a) with Figure 6(a). Similarly, an ideal current source set at $0A$ will correspond to an open connection: compare Figure 8(b) with Figure 6(b). This concept helps understanding the substitution of sources in Thevenin and Norton theorems.

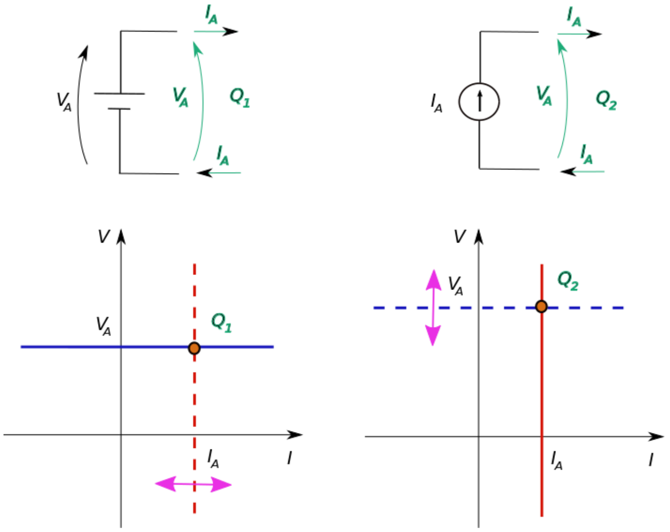


Figure 7: The ideal voltage and current sources interacting with other external circuitry (generic, not shown) through respective interfacing voltage V_A and current I_A.

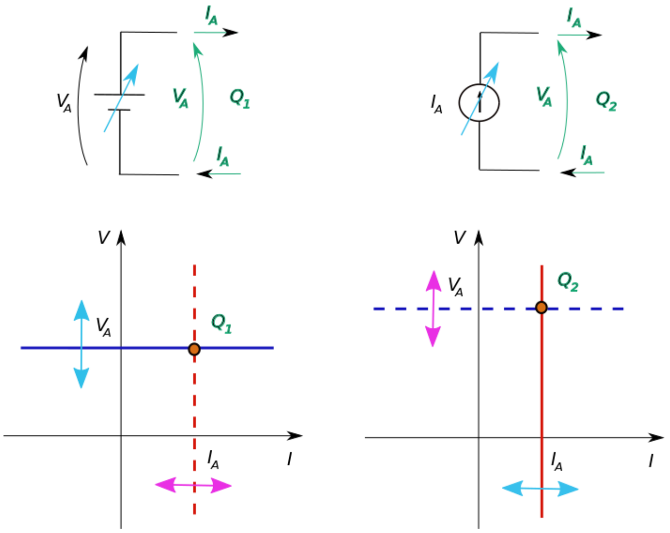


Figure 8: The adjustable (or variable) ideal voltage and current sources allow the respective fixed supplied voltage and current to be varied (blue arrows).

7 Voltage and Current Sources with Internal Resistance

Voltage and current sources incorporating internal resistances are illustrated in Figure 9.

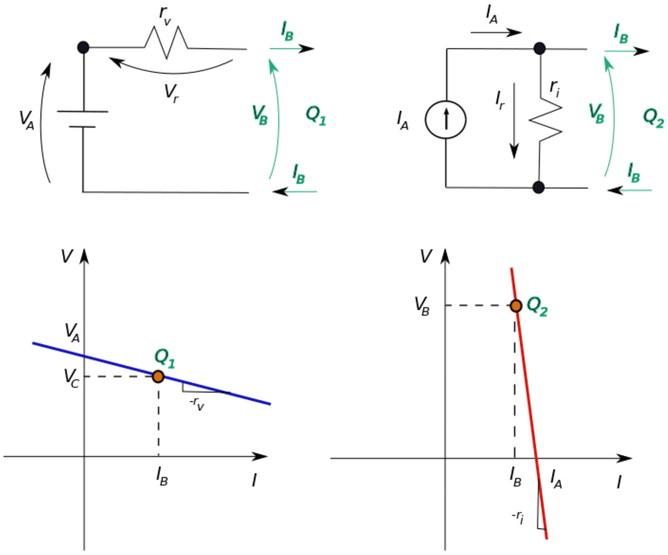


Figure 9: Voltage and current sources incorporating internal resistances, respectively r_v and r_i, and the corresponding characteristic curves.

In the case of the voltage source, the internal (unwanted) resistance is represented as being in series with the ideal voltage source, therefore implying, when current is supplied, a reduction of the expected voltage V_A corresponding to $r_v I_B$, so that $V_B < V_A$. Observe that thisvoltage source is still not real, in the sense that it can supply infinite power, etc.

A similar situation is verified for the current source with inner resistance. Here, the internal (unwanted) resistance is in parallel with an ideal current source, therefore deviating part of the current I_A that would be expected (a kind of leakage).

Interestingly, the inclination of both the curves in Figure 9 correspond to the negative value of the respective internal resistances, i.e. $-r_v$ and $-r_i$. As a consequence, if we make $r_v = r_i$, *the two sources effectively become electrically identical*. This establishes an interesting and useful equivalence between the voltage and current sources with internal resistances.

8 A Graphic Method

Given two devices (dipoles) represented by identical interfacing voltages and currents (same names and directions) V_A and I_A, it is possible to obtain the respective graphical solution, i.e. to determine the values of V_A and I_A when the two devices are connected.

The solution corresponds to the possible intersection between the two respective characteristic curves, which defines a respective operation point Q, unless there are problems with the design of the theoretical circuit. Situations where no intersection can be found should never be used.

The graphical solution method is illustrated in Figure 10.

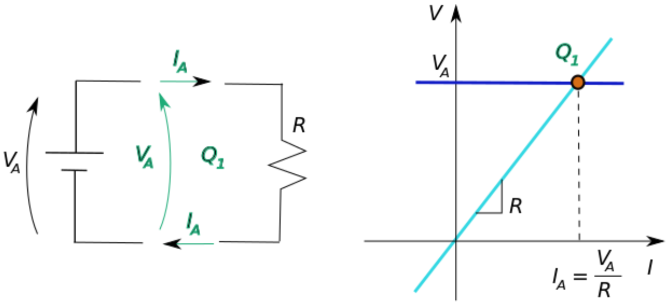


Figure 10: Example of applying the graphic method to determine the solution of the electrical configuration obtained when two devices (ideal voltage source and resistor in this illustration) with respective interfacing voltage and current V_A and I_A are interconnected. The solution corresponds to the intersection between the two characteristic curves, which defines a respective operation point Q_A. Situations leading to no intersection correspond to short circuits and need to be completely avoided.

Figure 11 illustrates another application of the graphic method to solving a circuit configuration corresponding to two ideal resistors connected in series.

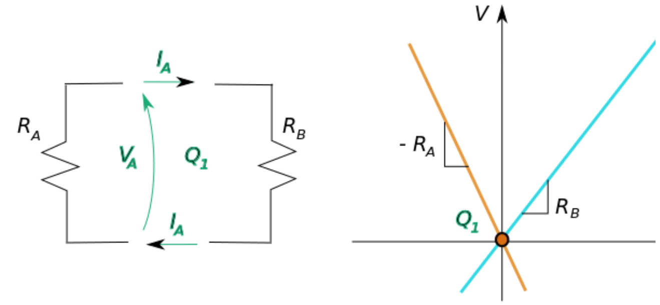


Figure 11: Graphical solution of a circuit consisting of two ideal resistors in series. Observe that the characteristic curve respective to resistor R_A, which has current direction corresponding to the inverse of the reference, was inverted with respect to the current axis. As expected, the solution yields the operation point ($V = 0, I = 0$).

Different types of connections between open and short connections is illustrated in Figure 12. Observe that these situations correspond to the previous series coupling of two resistors when the resistances are taken to the zero and infinite limits.

In the first case, we end up with a complete circuit corresponding to a perfect conducting wire at operation

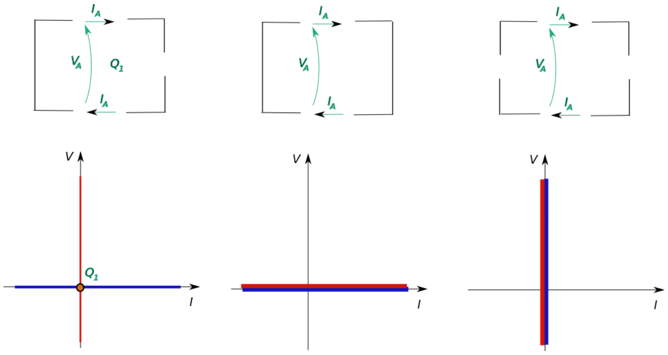


Figure 12: Three possible interconnections between open and short connections.

point ($V = 0, I = 0$). In the second case, when two short connections are connected, we obtain an ideal closed conducting loop, which can have any current at zero voltage. In the third case, corresponding to the interface between two open connections, we obtain two disconnected ideal wires.

As will be briefly discussed later in this work. the graphic method, which provides an interesting means to enhance our intuition and understanding of electrical circuits behavior, can also be used with some types of nonlinear devices (e.g. diodes, transistors).

9 Linear Superimposition

Given a linear circuit with more than one ideal voltage or power source, it is often possible to simplify its solution, in the sense of determining the voltage and current at every dipole device by applying the *linear superimposition* principle. This method is a direct consequence of the assumed linear nature of the circuits.

The method is simple and involves solving the system while considering only one of the ideal voltage or current sources at each time, and then adding the obtained results considering the respective directions. When ideal voltage sources are to be ignored, they are set to $0V$ and therefore become short connections (see Section 6). When an ideal current source is to be overlooked, it is set to $0A$ and becomes an open connection.

The solution of a particular circuit by using the linear superimposition method is illustrated in Figure 13.

The circuit to be solved is shown in (a). First, the ideal current source is set to $0A$, and is therefore substituted for an open connection (b). This results in only one mesh (right-hand side), which consists of a voltage divider with equal resistances of 300Ω. The $3V$ produced by the ideal voltage source is equally divided between these two resistances, yielding $5mA$. Then, the ideal voltage source is substituted by a short connection (c), resulting in the two identical resistances of 300Ω resulting in parallel configuration one another, therefore dividing the $2mA$ current produced by the ideal current source into two branches with equal resistances of 300Ω. Each of the $1mA$ acting on these resistances result in respective voltages of $0.3V$. The $2mA$ current through the 500Ω resistor implies $1V$ voltage on this resistor. The voltages and currents obtained from the two analyses are them superimposed (added) considering the directions and signals, yielding the result in (d).

10 Thevenin and Norton Equivalent Circuits

Given a linear circuit composed of ideal components, and any two distinct points in this circuit delimiting what we will call an *inner* part of the circuit (e.g. a dipole or a sub-circuit connected to the remainder circuit through two points) in which we are particularly interested, it is possible to substitute the entire external circuit by an equivalent circuit composed of only a ideal voltage or current source and an ideal equivalent resistance, which are called the *Thevenin and Norton* (respectively) *equivalent circuits*. These concepts are particularly interesting when combined with the above described graphic analysis.

First, the 'innerl' part of the circuit between the two selected points is removed from the original circuit, and the voltage between that pair of points is determined by using methods for circuit solution that can include the linear superimposition principle. This yields the *Thevenin voltage* V_{Th}.

Then, with the 'inner' portion removed from the main circuit, all ideal voltage sources are set to $0V$, yielding short connections, and all ideal current sources are set to $0A$, being substituted by open connections. The equivalent resistance of the resulting circuit composed only by resistances (or, eventually, conductances) is then determined by using standard methods, yielding the *Thevenin resistance* R_{Th}.

Figure 14 presents an example of Thevenin equivalent circuit determination with respect to the circuit in (a) and considering as the 'inner' portion of the circuit the resistor within by the dashed box, defining the two points of reference A and B. The remainder of the circuit is understood as being 'external', and it is this part of the original circuit that is to be reduced to its equivalent Thevenin circuit.

After removing the resistor inside the dashed box, the voltage V is calculated, for instance by using the superimposition method, as corresponding to $V = 2.4V$ from point A to B. The Thevenin equivalent resistance is calculated by replacing, in the circuit in Figure 14, the ideal voltage source by a short connection and the ideal current

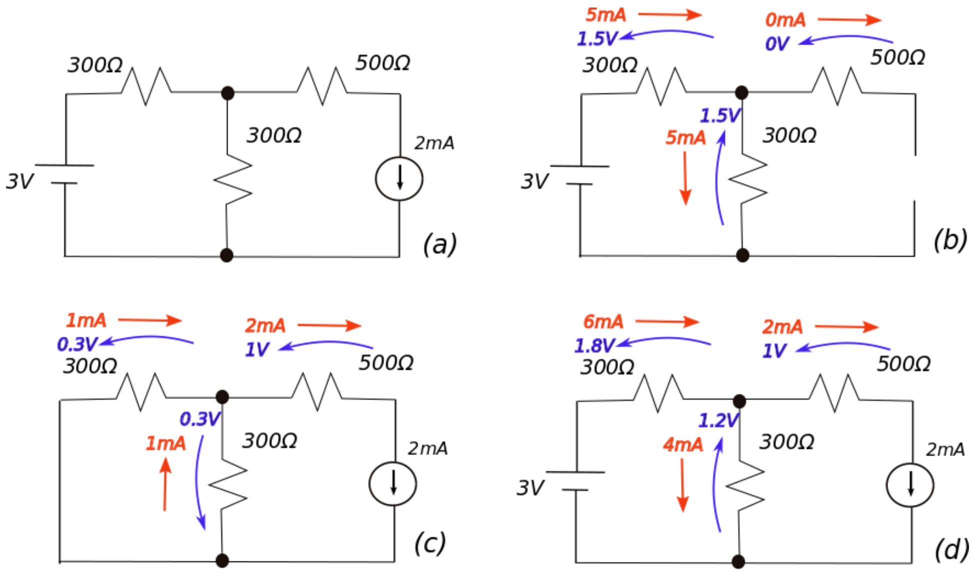


Figure 13: Example of solution of a circuit, shown in (a) by using the linear superimposition method. First, the ideal current source is set to $0A$, becoming an ideal open connection (b), and the voltages and currents at each dipole device are calculated by using standard methods of linear DC circuits. Then, the ideal voltage source is set to $0V$, becoming an ideal short connection (c), and the respective voltages and currents are determined along the circuit. The solution can be obtained by adding the previously derived voltages and currents respectively, taking into account their directions/signals, yielding the result shown in (d).

source by an open connection, yielding $R = 300\Omega$.

The initial internal portion, i.e. the $r00\Omega$ resistor, can now be reconnected to the terminas A and B and the respective voltage and current be determined. It is left as an exercise to calculate these values and compare them with those obtained in the example in Section 9.

Having obtained the Thevenin equivalent circuit, the respective Norton equivalent circuit can be easily determined by using the same resistance R_{Th} and making $I_{Th} = V_{Th}/R_{Th}$ (see Section 7).

11 Some Non-Linear Devices

Though the graphic method described above assumed the circuit to be linear, it can be applied to analyze some types of non-linear devices, such as non-linear resistors, diodes, and Zener diodes. The main exception concerns devices with *memory* (e.g. hysteresis, Gunn and tunnel diode, etc.), which imply voltage× current characteristic curves which cannot be represented as invertible functions (e.g. a same point in the x-axis mapping into two or more points in the y-axis when considering current as input, or vice versa when the voltage is considered as input).

Figure 15 illustrates a non-linear resistor attached to an ideal voltage supply. Though, in principle every real-world resistor is non-linear, they are often considered as operating in a linear regime. However, several types of resistors such as lamp filaments, will present a more pronounced linearity, typically in the sense that the resistance tends to increase with the current. In many cases, that is so because higher currents imply more heat dissipation, and in metallic conductors this will lead to more resistance. Interestingly, semiconductors can exhibit the opposite behavior, in the sense that higher temperature can reduce their resistance.

Diodes correspond to another important type of non-linear devices. The ideal diode involves no internal resistance, implying an abrupt transition from no-current to infinite current. Here we present a slightly more complete model incorporating an internal resistance r, as shown in Figure 16. Observe that the characteristic coordinates are often exchanged (i.e. current becomes the horizontal axis) when dealing with diodes and transistors, reflecting an emphasis on current-based action.

The diode acts as a short connection for voltages below a critical value V_c (approximately $0.7V$ for silicon-based diodes). As the source voltage V_A increases past V_c, the diode starts conducting current as a resistor with a typically low resistance r. Observe that the smaller the value of r, the less current variation one gets when varying the respective voltage.

Zener diodes are typically used as a relatively stable (for very small current) voltage reference. Figure 17 illustrates the graphical analysis of a Zener diode with internal resistance r (not shown) connected to an ideal voltage source.

Observe that the Zener diode is placed in opposite direction as the diode in Figure 16, as its voltage reference happens for negative current, where the device starts con-

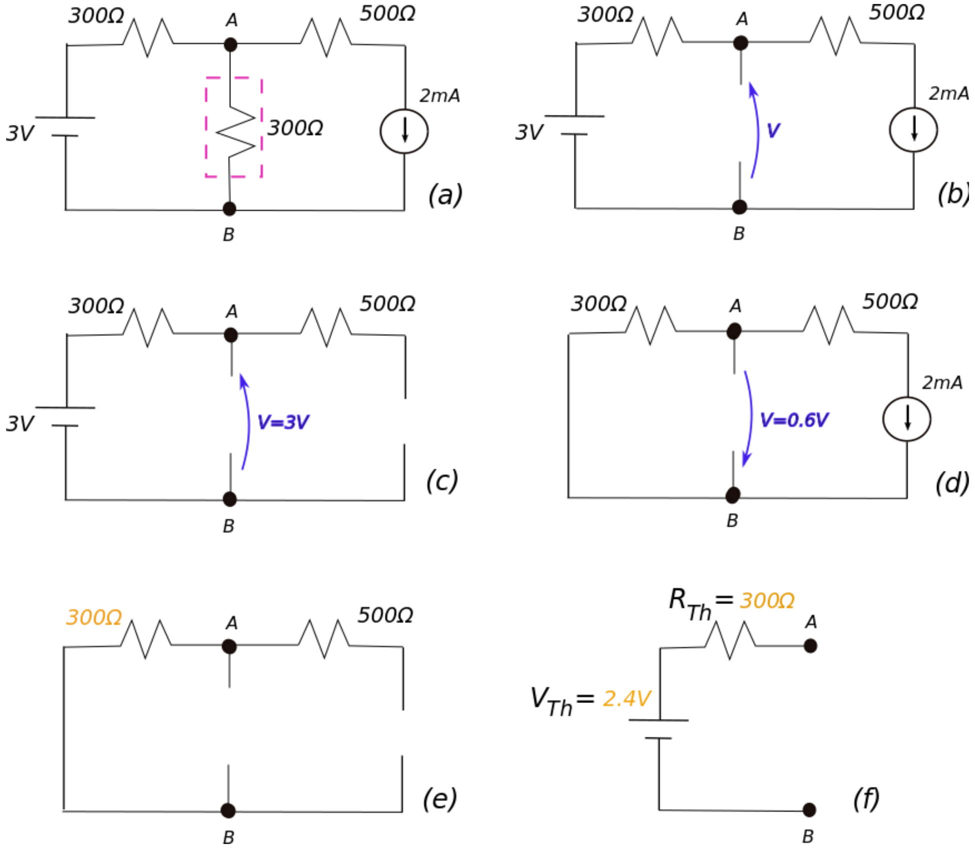


Figure 14: Determination of the Thevenin equivalent circuit for the portion of the circuit in (a) outside the dashed box (violet). First, the part inside the box is removed from the circuit (b). In order to determine the voltage between reference points A and B, we apply the superimposition principle to the circuit in (b), first considering the ideal voltage source (c), then the ideal current source (d), and the respective voltage values are then combined to yield $V = 2.4V$ from A to B. Then, the ideal voltage source is replaced by a short connection, and the ideal current source by an open connection, and the equivalent resistance of the remainder circuit is calculated as being equal to 300Ω (e). The resulting Thevenin equivalent is depicted in (f).

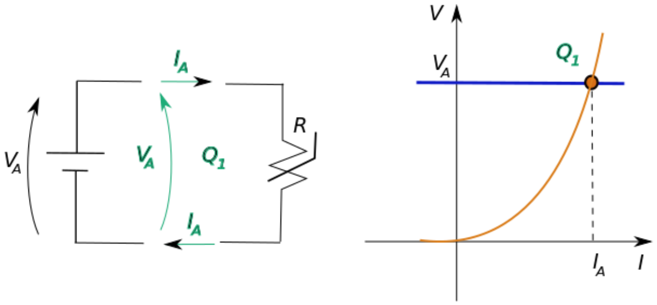


Figure 15: A simple circuit involving a non-linear resistor coupled to an ideal voltage source. Observe the deviation from the straight line expected from an ideal resistor (e.g. Fig. 3)

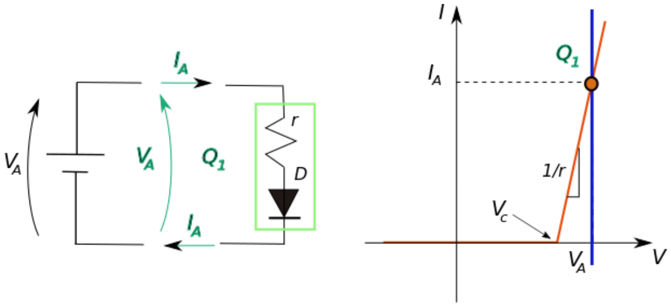


Figure 16: A diode with internal resistance connected to an ideal voltage supply.

ducting current as a resistor with internal resistance r.

12 Transfer Functions

In addition to providing the basis for the graphical solution of a circuit, the voltage × current characteristic curve has an additional advantage in the sense that it can also be understood as the *transfer function* of the respectively described device.

This possibility is illustrated in Figure 18 with respect to the characteristic of a non-linear device whose characteristic curve is shown in orange.

Figure

The action of the device on the relationship voltage × current can be graphically obtained by considering that

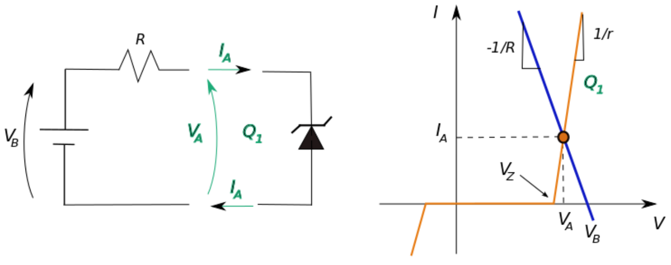


Figure 17: A Zener with internal resistance r (not shown) connected to an ideal voltage source in series with an ideal resistor R.

the characteristic curve actually corresponds as a funciton, i.e.:

$$V = f(I) \tag{2}$$

This implies that the resulting voltage can be obtained graphically by projectiong, one-by-one, the points of the current input signal $i_in(t)$ into the curve, so as to obtain $v_out(t)$, as illustrated in Figure 18. A similar situation is defined with respect to taking the voltage as input and current as output, in which case it is more intuitive to exchange the axes.

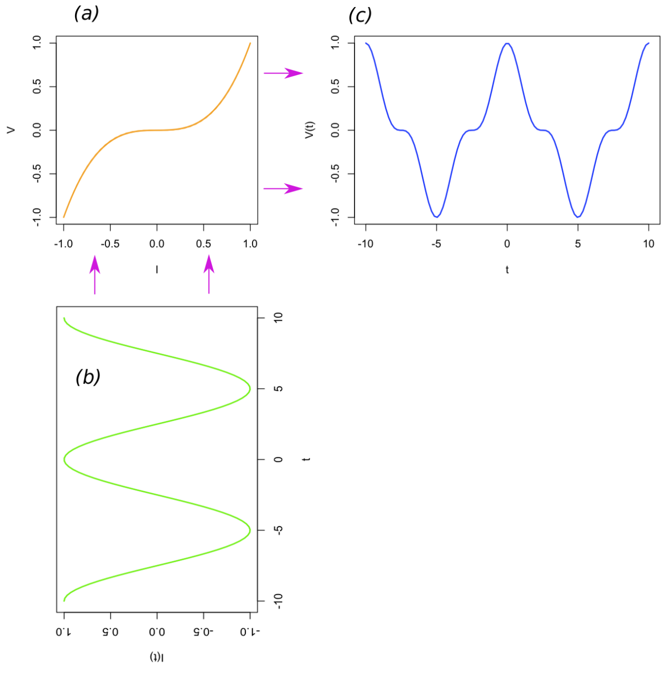


Figure 18: The characteristic curve of a device (a) can also be used to obtain a graphical representation of its effect, as a *transfer function*, on an input current signal (b), yielding an output voltage signal (c).

This method provides a valuable resource in the sense that the effect of the non-linearities can be readily appreciated from both the characteristic curve as well as the relative modifications between the input and output signal. As such, this method is particularly suitable for intuitive studies of non-linear devices such as diodes, transistors, etc.

13 Concluding Remarks

The graphical methods of circuits analysis based on the concept of characteristic curve has been briefly presented, discussed and illustrated, considering their potential applications in studying and characterizing electronic circuits.

We followed a progressive presentation, in which each of the basic components – namely ideal voltage and current sources as well as the ideal resistor, were presented and individually characterized in terms of respective characteristic curves. The important aspect of the voltage and current interface was emphasized, as this provide the key means through which two parts of a circuit, characterized by respective characteristic curves, can be theoretically coupled.

The practical concepts of linear superimposition, as well as Thevenin and Norton theorems, were also briefly presented and discussed from the perspective of the considered graphical approach, with which they integrate effectively. Then, we proceeded to discuss the graphical solution of linear circuits in terms of the superimposition of respective characteristic curves.

Though focusing on ideal linear devices, we also briefly addressed how the described methodology can also be applied also to non-linear devices, illustrating its application to ideal non-linear resistors and diodes.

Acknowledgments.
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It is hoped that CDTs can also incorporate new insights and analogies concerning the reported concepts and methods. We hope these characteristics will contribute to making CDTs interesting both to beginners as well as to more senior researchers.

Each CDT focuses on a limited set of interrelated concepts. Though attempting to be relatively self-contained, CDTs also aim at being relatively short. Links to related material are provided in order to complement the covered subjects.

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